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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2016

FIRST YEAR [BATCH 2015-18]

Date : 23/05/2016 **MATHEMATICS (General)**

Time : 11 am – 2 pm **Paper : II** Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

Answer any three questions from question nos. 1 to 5 : [3×5]

1. If by a rotation of axes about the origin, (ax+by) and (cx+dy) be changed to $(a'x'+b'y')$ and $(c'x'+d'y')$ respectively, then show that $ad-bc=a'd'-b'c'$. [5]

2. Show that the straight line $x+y+1=0$ and the two straight lines $(x+y)^2-3(x-y)^2=0$ form the sides of an equilateral triangle. [5]

3. Reduce the equation $4x^2-4xy+y^2+2x-26y+9=0$ to its canonical form and hence determine the nature of the conic. [5]

4. The polar of the point P w.r.t the circle $x^2+y^2=a^2$ touches the circle $4(x^2+y^2)=a^2$. Show that the locus of P is $x^2+y^2=4a^2$. [5]

5. Prove that the two conics $\frac{\ell_1}{r}=1-e_1\cos\theta$ and $\frac{\ell_2}{r}=1-e_2\cos(\theta-\alpha)$ will touch one another, if $\ell_1^2(1-e_2^2)+\ell_2^2(1-e_1^2)=2\ell_1\ell_2(1-e_1e_2\cos\alpha)$. [5]

Answer any three questions from questions nos. 6 to 10 : [3×5]

6. a) Show that the following vectors are coplanar $3\vec{a}-7\vec{b}-4\vec{c}, 3\vec{a}-2\vec{b}+\vec{c}$, $\vec{a}+\vec{b}+2\vec{c}$ where $\vec{a},\vec{b},\vec{c}$ are any three non-coplanar vectors. [2]

   b) Show that the three vectors $2\vec{i}-\vec{j}+\vec{k}$, $\vec{i}-3\vec{j}-5\vec{k}$, $3\vec{i}-4\vec{j}-4\vec{k}$ form the sides of a right-angled triangle. [3]

7. In a triangle ABC prove that $c^2=a^2+b^2-2ab\cos C$, where symbols have their usual meaning. [5]

8. a) If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors, then find the value of $(\vec{a}-\vec{b})\cdot(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})$. [2]

   b) Prove by vector method that if two medians of a triangle be equal then the triangle is isosceles. [3]

9. Find the vector equation of the plane through the point $(4,3,-1)$ and perpendicular to the vector $(3\hat{i}-4\hat{j}+\hat{k})$. Find also the distance of the point (1,2,3) from the plane. [5]

10. a) Find the torque about the point $B(3,-1,3)$ of a force P is $(4\vec{i}+2\vec{j}+\vec{k})$ which is passing through the point A(5,2,4). [2]

    b) A particle being acted on by a constant forces $(4\vec{i}+\vec{j}-3\vec{k})$ and $(3\vec{i}+\vec{j}-\vec{k})$ is displaced from the point $(\vec{i}+2\vec{j}+3\vec{k})$ to the point $(5\vec{i}+4\vec{j}-\vec{k})$. Find the total work done by the force. [3]

Group – B

Answer any five questions from questions nos. 11 to 18 : [5×5]

11. Show that if $x_n=\frac{3n+1}{n+2}$, then the sequence $\{x_n\}$ is strictly monotonic increasing. Is the sequence convergence? Justify your answer. [2+3]

12. a) Test for convergence of the series $\frac{1}{3\cdot 7}+\frac{1}{4\cdot 9}+\frac{1}{5\cdot 11}+\frac{1}{6\cdot 13}+\ldots.$ [3]

b) Is the following series convergent?

$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots.$ [2]

13. a) Show that between any two roots of $e^x\cos x=1$, there exists at least one root of $e^x\sin x-1=0$. [2]

b) Verify Lagrange's mean value theorem for the function $f(x)=x(x-1)(x-3)$ in [0,4]. [3]

14. Find a, b such that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$. [5]

15. Find the maximum value of $\left(\frac{1}{x}\right)^x$. [5]

16. Use the Lagrange's method of undetermined multipliers to find the minimum value of $x^2+y^2+z^2$ subject to the condition x+y+z = 6. [5]

17. Find the asymptotes of the curve $y^3-xy^2-x^2y+x^3+x^2-y^2-1=0$. [5]

18. Find the envelope of the family $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where the parameters a & b are connected by a+b = c. [5]

Answer <u>any two</u> questions from question nos. <u>19 to 21</u> : [2×5]

19. Evaluate $\int_0^1 \cot^{-1}(1-x+x^2)dx$. [5]

20. Find the Reduction formula for $\int_0^{\pi/4}\tan^n x\,dx$, n being a positive integer. Use it to evaluate $\int_0^{\pi/4}\tan^6 x\,dx$. [5]

21. Evaluate the following limit : $\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\ldots\left(1+\frac{n}{n}\right)\right\}^{1/n}$. [5]

Answer <u>any two</u> questions from question nos. <u>22 to 24</u> : [2×5]

22. a) Find the order and degree of the following differential equation : $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{2/3}=\frac{d^2y}{dx^2}$. [2]

b) Formulate the differential equation from $y=e^{-x}(A\cos x+B\sin x)$ (A, B : parameters) [3]

23. Find the integrating factor and the general solution of the equation $(y^2e^x+2xy)dx-x^2dy=0$. [5]

24. Find the general and singular solution of $y=px+\sqrt{a^2p^2+b^2}\left(p\equiv\frac{dy}{dx}\right)$. [5]

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